PERFORMANCE METRICS OF THE MULTISTAGE INTERCONNECTION NETWORKS

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Abstract

*The performance of the multiprocessor system depends upon the performance of the MIN. The various parameters Adams G.B. et al.(1987);Sengupta J(2005) that are evaluated for analyzing the MIN are*Bandwidth (BW),Probability Of Acceptance (Pa),Processor Utilization (PU) ,Processing Power (PP),Throughput ,Cost,Fault-Tolerance,Permutations passable and Reliability

Keywords Active Memory modules, Output rate, Fault tolerance, Identical permutations and Incremental permutations.

Bandwidth

It is defined as average number of active memory modules in a transfer cycle of the Interconnection Networks.

Let p be the request rate at each of the a inputs of an a*b cross-bar module, the expected number of request that it passes per unit time is calculated as

$$b-b(1-p/b)^a$$

Dividing the above expression by the number of output lines of the a * b module gives the rate of requests on any one of the b output lines. Thus for any stage of a MIN, the output rate of request q is a function of its input rate which is given as

$$1-(1-p/b)^a$$

$$q=1-(1- p/b)^a$$

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Since the output rate of a stage is the input rate of the next stage, output rate of any stage can be recursively evaluated starting at stage 1. In particular, the output rate of the final stage n determines the bandwidth of a MIN. Let q_i be defined as the rate of requests on an output link of stage i, the following equation determines the bandwidth of an $a^n * b^n$ MIN.

Bandwidth (BW) = $b^n * P_n$,where P_n is the probability of requests being forwarded & P_0=P.

Probability of acceptance

It is defined as the ratio of bandwidth to the expected no of requests generated per cycle.

$$Pa=BW/a^n * p\ req\_gen$$

Processor utilization

It is defined as percentage of time the processor is active doing computation without accessing the global memory.

$$PU=BW/a^n * p\ req\_gen *T$$, where T is average time used for a memory or read/write operation.

Processor power

It is defined as sum of processor utilization over the number of processors.

$$PP=a^n*PU$$

Throughput

It is defined as maximum amount of data delivered per unit time.

$$Throughput = PU * p\ req\_gen$$

Cost

The cost of a network is calculated by considering the complexities of the components used. For a switch the cost is proportional to the no of gates counts i.e. no of cross points within a switch. The cost of multiplexers and demultiplexers is calculated in the same way. For example, n:1 multiplexer has n units of cost and 1: n demultiplexer has same units of cost i.e. n units of cost.

Fault-Tolerance

If the network is able to work, of course with degraded efficiency, in the presence of faults in critical components then the network is called as Fault-Tolerant. A network is single Fault-Tolerant if it can work with full access in the presence of fault in single SE. If the network is able to provide connections from all sources to all destinations in the presence of k faults in the network, then this network is called as k Fault-Tolerant network.

Permutations passable

It is a set of N data transfers, all of which are performed simultaneously in the network. The log2N stages network allows only some subset of N! passable permutations.

Let the source and destination in binary be represented as

$$S=S_{n-1}\ldots\ldots\ldots\ldots S_1S_0$$

$$D=D_{n-1}\ldots\ldots\ldots\ldots D_1D_0$$

There are two ways to evaluate the Permutations passable of the MINs: Identical permutations and Incremental permutations.

Identical Permutations is one to one communication between same source and same destination.

Example $S_{n-1} \rightarrow D_{n-1}$, --------- , $S_o \rightarrow D_o$

In Incremental Permutations,each source is connected to destination in a circular chain.

Example $S_0 \rightarrow D_4$, ……… , $S_{n-1} \rightarrow D_3$

The presence of faults in the network is considered under two cases: Non-critical (n-cr) and Critical (cr). Non-critical is a case when fault is present in a single switch. Critical is a case when fault is present in a loop. Permutations passable should not get effected under faulty cases other than the critical faults.

MTTF

Reliability R(t) is the probability that the network does not fail in the interval (0,t).The network is assumed to be faulty if any source destination pair cannot be connected because of the presence of faulty components in the network.

The reliability can be measured in terms of MTTF i.e. Mean Time To Failure Balaguruswami E (1989). It is defined as expected time elapsed before some source is disconnected from some destination.

Some of the assumptions used to calculate the reliability are as under

1. Switches are statistically identical and are either fully operational or failed.
2. The Switch failure occur with a failure rate of $\lambda = 10^{-6}$per hour.
3. Failure rate of 2 * 2 Switching Element is considered as $\lambda_2 = \lambda$
4. Failure rate of 3 * 3 Switching Element is taken as $\lambda_3 = 2.5\ \lambda$
5. For a m:1 multiplexer, the failure rate is considered as $\lambda_m = m\lambda/4$.
6. For 1:m demultiplexer, the failure rate is considered as $\lambda_d = \lambda_m$.
7. Switching Elements in the last stage and corresponding demultiplexers are taken together as a series system having failure rate of $\lambda_{2d} = 2\ \lambda$.

a) Upper Bound analysis

The network is operational if the critical set of switches is operational. The critical set is the set of k SEs, each from different module such that a failure occurs if all k SEs are faulty simultaneously Blake J.T et al.(1989). The expression for Upper Bound Reliability is

$$MTTF = \int_0^\infty R_{UB}\ (t)\ dt$$

b)Lower Bound analysis

In the Lower Bound Analysis, the input side SEs and their corresponding multiplexers are considered as a series system and failure of any component leads to the failure of all three.

The expression for Lower Bound Reliability is

$$MTTF = \int_0^\infty R_{LB}\ (t)\ dt$$

The probability, Rcs(t) of a critical set not being faulty is

$$R_{cs}(t) = [\ 1\text{-}[1\text{-}e^{\lambda t}]^{m}]$$

A good IN should have good performance i.e. high values of Bandwidth, Probability of Acceptance, Processor Utilization, Processor Power and Throughput. It should have lower values of path length and should be cost effective. In this type of network the Permutations passable should not get effected under faulty cases other than critical faults. The network should be reliable.

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